

# Five-dimensional Massive Vector Fields and Radion Stabilization

Emilian Dudas <sup>a,b</sup> and Mariano Quiros <sup>c</sup>

<sup>a</sup> *Centre de Physique Théorique<sup>†</sup>, Ecole Polytechnique, F-91128 Palaiseau, France*

<sup>b</sup> *LPT<sup>‡</sup>, Bât. 210, Univ. de Paris-Sud, F-91405 Orsay, France*

<sup>c</sup> *Institució Catalana de Recerca i Estudis Avançats (ICREA)*

*Theory Physics Group, IFAE/UAB, E-08193 Bellaterra, Barcelona, Spain*

## Abstract

We provide a description of the five-dimensional Higgs mechanism in supersymmetric gauge theories compactified on the orbifold  $S^1/\mathbb{Z}_2$  by means of the  $\mathcal{N} = 1$  superfield formalism. Goldstone bosons absorbed by vector multiplets can come either from hypermultiplets or from gauge multiplets of opposite parity (Hosotani mechanism). Supersymmetry is broken by the Scherk-Schwarz mechanism. In the presence of massive hypermultiplets and gauge multiplets, with different supersymmetric masses, the radion can be stabilized with positive (de Sitter) vacuum energy. The masses of vector and hypermultiplets can be fine-tuned to have zero (Minkowski) vacuum energy.

---

<sup>†</sup>Unité mixte du CNRS et de l'EP, UMR 7644.

<sup>‡</sup>Unité mixte du CNRS, UMR 8627.

## 1. Introduction

Some time ago it was shown [1] in the context of Scherk-Schwarz type compactifications [11] that massive hypermultiplets in five dimensions have a contribution to the one-loop vacuum energy such that, when added to the one-loop gravitational contribution, they do stabilize the radion field with a resulting negative vacuum energy. The goal of the present paper is to revisit this mechanism of radion stabilization in five dimensions in theories where massless and massive hypermultiplets, as well as massive vector multiplets are present. As we will prove, if hypers and vectors have different 5D masses, the vacuum energy has different contributions with different signs and the resulting vacuum energy can generate a radion stabilization with positive (de Sitter) or zero (Minkowski) vacuum energy.

Along the way, we provide a description of the five-dimensional Higgs mechanism in supersymmetric gauge theories. Long time ago Fayet [2] presented the Higgs mechanism for  $\mathcal{N} = 2$  supersymmetric theories in four dimensions and showed that a vector multiplet can become massive in two different ways. The first way corresponds to the Goldstone boson coming from a hypermultiplet and requires a minimal number of one vector and one hypermultiplet to start with. It corresponds to spontaneous symmetry breaking when the scalar component of a hypermultiplet acquires a vacuum expectation value. The simplest realization is the case of a  $U(1)$  gauge theory with one charged hypermultiplet. The second way corresponds to the Goldstone boson coming from a vector multiplet and requires more than one vector multiplet to begin with, i.e. it can only be realized in non-abelian gauge theories. We will see that it corresponds to the Hosotani breaking [3], the simplest case being an  $SU(2)$  gauge theory. We provide the uplift of these two mechanisms in five dimensions in the superfield formalism [4, 5, 6].

In Section 2 we present the Higgs mechanism in a 5D theory compactified on an orbifold  $S^1/\mathbb{Z}_2$  for a  $U(1)$  gauge field, with a Goldstone coming from a charged hypermultiplet, in a non-linear realization of the gauge symmetry, more convenient in describing the physical spectrum. We show that the quadratic part of the action is already 5D and gauge invariant and contains a natural generalization of Stueckelberg couplings. In Section 3 we discuss the 5D Higgs mechanism for a non-abelian gauge field, in which a  $U(1)$  generator of an  $SU(2)$  gauge group absorbs a Goldstone coming from another vector multiplet, again using a convenient non-linear realization of the gauge symmetry, which parametrizes a vacuum expectation value for the fifth component of a gauge field (Hosotani mechanism). In this case, the perturbative lagrangian in the vacuum spontaneously breaks the 5D Lorentz in-

variance. In Section 4 we discuss the appropriate Scherk-Schwarz supersymmetry breaking deformation, taking into account the different spontaneous breaking of the  $R$ -symmetry group in the two cases. In Section 5 we consider massive hypermultiplets and massive vector multiplets in five dimensions, of different masses, in the presence of Scherk-Schwarz supersymmetry breaking. We show that the one-loop vacuum energy induced by supersymmetry breaking generates a potential for the radion which, depending on the number of massless and massive hyper and vector multiplets and on their masses, can stabilize it with a positive or zero vacuum energy, for both possible Higgs patterns of symmetry breaking. We also provide a four-dimensional description of the stabilization mechanism in terms of one-loop corrections to the 4D Kahler potential.

## 2. Higgs mechanism in five dimensions with a Goldstone coming from a hypermultiplet

The framework we are considering is a five dimensional supersymmetric gauge theory compactified on the interval  $S^1/\mathbb{Z}_2$ , leaving one unbroken supersymmetry in four dimensions. The fifth coordinate  $x^5$  goes from 0 to  $\pi L$ . Let us consider one five dimensional vector multiplet, described in 4D superfield language [4, 5, 6] by  $(V, \Phi)$  with  $\mathbb{Z}_2$  parities  $(+, -)$  and one hypermultiplet  $(T_1, T_2)$  of  $\mathbb{Z}_2$ -parities and  $U(1)$ -charges  $(+, -)$ . The invariant action under the  $U(1)$  symmetry is <sup>1</sup>

$$\begin{aligned} S = & \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha + T_1 \left( \partial_5 - \frac{\Phi}{\sqrt{2}} \right) T_2 \right] + \text{h.c.} \\ & + \int d^4\theta \left[ \left( \partial_5 V - \frac{\Phi + \bar{\Phi}}{\sqrt{2}} \right)^2 + T_1 e^V \bar{T}_1 + T_2 e^{-V} \bar{T}_2 - \xi V \right] \end{aligned} \quad (2.1)$$

where  $\xi$  is a Fayet-Iliopoulos parameter. In fact (2.1) is invariant under the gauge transformations

$$\begin{aligned} V &\rightarrow V + \Lambda + \bar{\Lambda}, \quad \Phi \rightarrow \Phi + \sqrt{2} \partial_5 \Lambda \\ T_1 &\rightarrow e^{-\Lambda} T_1, \quad T_2 \rightarrow e^{\Lambda} T_2 \end{aligned} \quad (2.2)$$

where  $\Lambda$  is an  $\mathcal{N} = 1$  chiral superfield <sup>2</sup>.

---

<sup>1</sup>We work in what follows in units where the gauge coupling is one  $g_5 = 1$ . An integral over the 5D spacetime is implicit in the action.

<sup>2</sup>With the present field content, the  $\mathbb{Z}_2$  orbifold induces a gauge anomaly at the orbifold fixed points. Consistency of the theory can be achieved in several ways. One possibility is to add a second bulk charged

If we want to describe the spontaneous breaking of the  $U(1)$  gauge symmetry when the scalar component of one of the chiral superfields (say  $T_1$ ) acquires a vacuum expectation value  $\sim \sqrt{\xi}$  it is convenient to change the superfield basis and realize non-linearly the gauge symmetry by introducing the superfields  $Z_{1,2}$  as

$$\begin{aligned} T_1 &= -\sqrt{2}m e^{-Z_1/\sqrt{2}} \\ T_2 &= Z_2 e^{Z_1/\sqrt{2}} \end{aligned} \quad (2.3)$$

where  $\xi = 2m^2$ . In this basis the corresponding gauge transformations in (2.2) are

$$Z_1 \rightarrow Z_1 + \sqrt{2}\Lambda, \quad Z_2 \rightarrow Z_2 \quad (2.4)$$

and therefore the Goldstone boson is in the scalar component of  $Z_1$ . In terms of the new fields, and expanding the action (2.1) in a power series of the gauge coupling, the quadratic part of the action describing the system is given by

$$\begin{aligned} S^{(2)} &= \int d^2\theta \left[ \frac{1}{4} W^\alpha W_\alpha + m(\Phi - \partial_5 Z_1) Z_2 \right] + \text{h.c.} \\ &+ \int d^4\theta \left[ \left( \partial_5 V - \frac{\Phi + \bar{\Phi}}{\sqrt{2}} \right)^2 + \bar{Z}_2 Z_2 + m^2 \left( V - \frac{Z_1 + \bar{Z}_1}{\sqrt{2}} \right)^2 \right] \end{aligned} \quad (2.5)$$

where the relationship between  $\xi$  and  $m$  has been used to cancel the linear term in  $V$ . The truncation to the quadratic part of the action is sufficient for the purpose of finding the physical spectrum, needed for computing the one-loop vacuum energy in Section 5.

We will now use the notation for the  $\mathcal{N} = 1$  field content of the different superfields:  $V \equiv (A_\mu, \lambda, D)$ ,  $\Phi \equiv (\phi, \psi_\phi, F_\phi)$  and  $Z_i \equiv (z_i, \chi_i, F_i)$ . We want to describe the abelian Higgs mechanism in five dimensions with the imaginary part of  $z_1$  being the Goldstone boson absorbed by the gauge field which becomes massive. We are describing the system directly in the Stueckelberg, non-linear realization of the gauge symmetry [9]. The KK hypermultiplet such that the surviving fermionic zero mode cancels the gauge anomaly. If this second hyper gets no vacuum expectation value, it is a spectator for the analysis of the present section and counts as an additional hyper in the computation of Section 5. A second possibility is adding a chiral multiplet localized on one of the fixed points in order to cancel the global anomaly. In this case the surviving local anomaly can be cancelled by a 5D Chern-Simons (CS) term in the bulk Lagrangian  $\sim A \wedge F \wedge F$  [7]. Neither the localized chiral multiplet nor the CS term are going to alter the analysis of this section. Still a third possibility is to use a second, bulk odd vector 5D superfield whose fifth component can shift anomalies in a 5D version of the Green-Schwarz mechanism [8].

gauge boson masses are immediately found out to be

$$m_A^{(k)^2} = m^2 + \frac{k^2}{L^2} . \quad (2.6)$$

The fermion masses are described by the mass terms

$$\begin{aligned} S_{\text{ferm}} &= \sum_k \left\{ -im\lambda^{(k)}\chi_1^{(k)} + i\frac{k}{L}\lambda^{(k)}\psi_\phi^{(k)} + \frac{k}{L}\chi_1^{(k)}\chi_2^{(k)} + m\psi_\phi^{(k)}\chi_2^{(k)} \right\} + \text{h.c.} \\ &\equiv V\mathcal{M}^{(KK)}V^T + \text{h.c.} \end{aligned} \quad (2.7)$$

with  $V \equiv (\lambda^{(k)}, \psi_\phi^{(k)}, \chi_1^{(k)}, \chi_2^{(k)})$  and  $\mathcal{M}^{(KK)}$  is the fermionic mass matrix. The eigenvalues of the fermionic mass matrix are

$$(\mathcal{M}^\dagger\mathcal{M})^{(KK)} = \left(m^2 + \frac{k^2}{L^2}\right) \mathbf{1} . \quad (2.8)$$

The canonically normalized Goldstone superfield is  $\hat{Z}_1 = mZ_1$  and the unitary gauge is  $Z_1 = 0$ . In what follows, for notational simplicity, we ignore the hat notation on the canonically normalised Goldstone field.

In order to check the full  $\mathcal{N} = 2$  supersymmetry in the spectrum we are also deriving the scalar mass matrix. By defining the scalar components

$$\phi \equiv \frac{\Sigma + iA_5}{\sqrt{2}}, \quad z_1 \equiv \frac{\phi_1 + ia_1}{\sqrt{2}}, \quad (2.9)$$

the scalar part of the lagrangian is given by

$$\begin{aligned} S_{\text{scalar}} &= |F_1|^2 + |F_2|^2 + |F_\phi|^2 + \frac{1}{2}D^2 - \Sigma\partial_5 D - mD\phi_1 \\ &+ \left[ (m\phi - \partial_5 z_1)F_2 + (mF_\phi - \partial_5 F_1)z_2 + \text{h.c.} \right] . \end{aligned} \quad (2.10)$$

By eliminating the auxiliary fields

$$\begin{aligned} D &= m\phi_1 - \partial_5 \Sigma \quad , \quad F_1 = -\partial_5 \bar{z}_2 \quad , \\ F_2 &= -m\bar{\phi} + \partial_5 \bar{z}_1 \quad , \quad F_\phi = -m\bar{z}_2 \end{aligned} \quad (2.11)$$

and performing some straightforward integrations by parts, we obtain the scalar potential

$$\begin{aligned} V &= m^2|z_2|^2 + |\partial_5 z_2|^2 + \frac{1}{2}[m^2\phi_1^2 + (\partial_5 \phi_1)^2] \\ &+ \frac{1}{2}[m^2\Sigma^2 + (\partial_5 \Sigma)^2] + \frac{1}{2}(mA_5 - \partial_5 a_1)^2 . \end{aligned} \quad (2.12)$$

By performing now the standard KK expansions

$$a_1 = \frac{1}{\sqrt{\pi L}} \sum_{k=0}^{\infty} r_k \cos \frac{ky}{L} a_1^{(k)} \quad , \quad A_5 = \frac{1}{\sqrt{\pi L}} \sum_{k=1}^{\infty} \sin \frac{ky}{L} A_5^{(k)} \quad , \quad (2.13)$$

where  $r_0 = 1/\sqrt{2}$  and  $r_k = 1$  for  $k = 1 \dots \infty$  and defining the mass eigenstates

$$\begin{aligned} \xi_1^{(k)} &= \frac{1}{\sqrt{m^2 + k^2/L^2}} \left[ m a_1^{(k)} - \frac{k}{L} A_5^{(k)} \right] \quad , \quad \text{for } k = 0, \dots, \infty \quad , \\ \xi_2^{(k)} &= \frac{1}{\sqrt{m^2 + k^2/L^2}} \left[ \frac{k}{L} a_1^{(k)} + m A_5^{(k)} \right] \quad , \quad \text{for } k = 1, \dots, \infty \quad , \end{aligned} \quad (2.14)$$

we find the mass lagrangian for KK states

$$\begin{aligned} S_{\text{scalar}} &= \sum_{k=1}^{\infty} \left( m^2 + \frac{k^2}{L^2} \right) \left[ |z_2^{(k)}|^2 + \frac{1}{2} (\Sigma^{(k)})^2 + \frac{1}{2} (\xi_2^{(k)})^2 \right] \\ &+ \sum_{k=0}^{\infty} \left[ \frac{1}{2} \left( m^2 + \frac{k^2}{L^2} \right) (\phi_1^{(k)})^2 + r_k \sqrt{m^2 + \frac{k^2}{L^2}} A_\mu^{(k)} \partial^{(\mu)} \xi_1^{(k)} \right] \quad . \end{aligned} \quad (2.15)$$

The physical interpretation is that  $\xi_1^{(k)}$  are the Goldstone bosons eaten by the KK modes of the gauge fields which become massive, while five scalar degrees of freedom  $z_2^{(k)}, \phi_1^{(k)}, \Sigma^{(k)}$  and  $\xi_2^{(k)}$  become massive with the same mass as the fermions (2.8). The three gauge field degrees of freedom plus the five massive scalar ones do correctly match the eight massive fermionic degrees of freedom in (2.8).

We would like now to check the 5D Lorentz invariance of the effective action. The fermionic part of the 5D lagrangian is best expressed by introducing the Dirac fermions

$$\Psi_1 = (\psi_\phi, -i\bar{\lambda})^T \quad , \quad \Psi_2 = (\chi_1, -\bar{\chi}_2)^T \quad . \quad (2.16)$$

Defining as usual the 5D gauge field  $A_M = (A_\mu, A_5)$  and combining the 4D gauge bosons and scalars terms, we find the total quadratic action

$$\begin{aligned} S^{(2)} &= -\frac{1}{4} F_{MN} F^{MN} - |\partial_M z_2|^2 - m^2 |z_2|^2 - \frac{1}{2} [(\partial_M \phi_1)^2 + m^2 \phi_1^2] \\ &- \frac{1}{2} [(\partial_M \Sigma)^2 + m^2 \Sigma^2] - \frac{1}{2} (m A_M - \partial_M a_1)^2 \\ &- i \bar{\Psi}_1 \gamma^M \partial_M \Psi_1 - i \bar{\Psi}_2 \gamma^M \partial_M \Psi_2 - m (\bar{\Psi}_1 \Psi_2 + \bar{\Psi}_2 \Psi_1) \quad . \end{aligned} \quad (2.17)$$

Notice the obvious generalisation of the Stueckelberg mixing between the gauge field  $A_M$  and the Goldstone boson  $a_1$  which appears in the last term of the second line of (2.17).

### 3. Higgs mechanism in five dimensions with a Goldstone coming from a vector multiplet

We will now consider the case of an  $SU(2)$  gauge theory broken into  $U(1)$  by the orbifold boundary conditions [10]. We then have three five dimensional vector multiplets, described in 4D superfield language [4, 5, 6] by  $(V_a, \Phi_a)$  with  $a = 1, 2, 3$  and  $\mathbb{Z}_2$  parities given in the Table.

$\mathbb{Z}_2$ -parity	+	-
Vector	$V_3$	$V_{1,2}$
Chiral	$\Phi_{1,2}$	$\Phi_3$

The action of the system can be written as [6]

$$\begin{aligned}
S = & \frac{1}{4} \int d^2\theta \operatorname{Tr} W^\alpha W_\alpha + \text{h.c.} \\
& + 2 \int d^4\theta \operatorname{Tr} \left[ \{e^{V/2}, \partial_5 e^{-V/2}\} + \frac{1}{\sqrt{2}} (e^{V/2} \bar{\Phi} e^{-V/2} + e^{-V/2} \Phi e^{V/2}) \right]^2
\end{aligned} \quad (3.1)$$

where  $V = V_a \sigma^a / 2$  and  $\Phi = \Phi_a \sigma^a / 2$ . In fact the action (3.1) is invariant under the supergauge transformations

$$V \rightarrow e^\Lambda e^V e^{\bar{\Lambda}}, \quad \Phi \rightarrow e^\Lambda (\Phi - \sqrt{2} \partial_5) e^{-\Lambda} \quad (3.2)$$

where  $\Lambda = \Lambda_a \sigma^a / 2$ .

We will now describe the spontaneous breaking of the surviving  $U(1)$  gauge symmetry by the Hosotani mechanism. The  $\mathcal{N} = 1$  notation is  $V_a = (A_a^\mu, \lambda_a, D_a)$ ,  $\Phi_a = (\phi_a, \psi_a, F_a)$  and  $\phi_a = (\Sigma_a + i A_a^5) / \sqrt{2}$ . We want to consider the Higgs mechanism where the imaginary part of  $\phi_1$ ,  $A_1^5$ , acquires a vacuum expectation value. Analogously to the first case, we are describing the system directly in the Stueckelberg, non-linear realization of the gauge symmetry by the field redefinition

$$\phi = \phi_3 \sigma^3 / 2 + e^{\chi_1 \sigma^3 / 2} (i \sqrt{2} m \sigma^1 / 2 + \chi_2 \sigma^2 / 2) e^{-\chi_1 \sigma^3 / 2} \quad (3.3)$$

in such a way that under the surviving gauge transformation  $\Lambda_3$  the fields transform as

$$\phi_3 \rightarrow \phi_3 - \sqrt{2} \partial_5 \Lambda_3, \quad \chi_1 \rightarrow \chi_1 + \Lambda_3, \quad \chi_2 \rightarrow \chi_2 \quad (3.4)$$

and the field  $\chi_1$  can be gauged away. In fact the relation between the parametrization in (3.3) and the  $\phi_a$  basis is given by

$$\begin{aligned}\phi_1 &= i(\sqrt{2}m \cosh \chi_1 - \chi_2 \sinh \chi_1) = i\sqrt{2}m + \mathcal{O}(\chi_1^2, \chi_1\chi_2) \\ \phi_2 &= \chi_2 \cosh \chi_1 - \sqrt{2}m \sinh \chi_1 = \chi_2 - \sqrt{2}m\chi_1 + \mathcal{O}(\chi_1^2, \chi_1\chi_2)\end{aligned}\quad (3.5)$$

The quadratic part of the  $D$ -term contribution to the action (3.1) can be written as

$$2 \int d^4\theta \text{Tr} \left[ \partial_5 V - \frac{\phi + \bar{\phi}}{\sqrt{2}} + \frac{1}{2\sqrt{2}} [V, \phi - \bar{\phi}] \right]^2$$

and in terms of the new fields the action (3.1) to quadratic order is

$$\begin{aligned}S^{(2)} &= \frac{1}{4} \int d^2\theta W_a^\alpha W_\alpha^a + \text{h.c.} \\ &+ \int d^4\theta \left\{ [\partial_5 V_1]^2 + \left[ \partial_5 V_3 + mV_2 - \frac{\phi_3 + \bar{\phi}_3}{\sqrt{2}} \right]^2 + \left[ \partial_5 V_2 - mV_3 - \frac{\phi_2 + \bar{\phi}_2}{\sqrt{2}} \right]^2 \right\}\end{aligned}\quad (3.6)$$

where the field  $\phi_2$  is defined as function of  $\chi_{1,2}$  as in Eq. (3.5). The action (3.7) is invariant under the gauge transformations

$$\begin{aligned}\delta V_3 &= \Lambda_3 + \bar{\Lambda}_3 \quad , \quad \delta \phi_3 = \sqrt{2} \partial_5 \Lambda_3 + \sqrt{2} m \Lambda_2 \quad , \\ \delta V_2 &= \Lambda_2 + \bar{\Lambda}_2 \quad , \quad \delta \phi_2 = \sqrt{2} \partial_5 \Lambda_2 - \sqrt{2} m \Lambda_3 \quad .\end{aligned}\quad (3.7)$$

Notice that at the quadratic order there is no gauge invariance corresponding to the  $V_1$  gauge boson, which is therefore a general real vector superfield. Since it is completely decoupled from the rest of the lagrangian and will play no role in our discussion, we ignore it in what follows.

In order to derive the scalar mass matrix, we start from the scalar part of the action

$$S_{\text{scalar}} = \frac{1}{2} (D_2^2 + D_3^2) - (\partial_5 D_3 + m D_2) \Sigma_3 - (\partial_5 D_2 - m D_3) \Sigma_2 \quad . \quad (3.8)$$

By eliminating the auxiliary fields

$$\begin{aligned}D_3 &= m \Sigma_2 - \partial_5 \Sigma_3 \quad , \\ D_2 &= -m \Sigma_3 - \partial_5 \Sigma_2\end{aligned}\quad (3.9)$$

and integrating by parts, we find the four-dimensional scalar potential

$$V = \frac{1}{2} \left[ (\partial_5 \Sigma_3)^2 + (\partial_5 \Sigma_2)^2 + m^2 \Sigma_3^2 + m^2 \Sigma_2^2 - 4m \Sigma_2 \partial_5 \Sigma_3 \right] \quad . \quad (3.10)$$



The full lagrangian in this case has spontaneously broken 5D Lorentz invariance due to the vacuum expectation value of the fifth component of  $A_1$ , whereas by construction the original lagrangian is clearly 5D Lorentz invariant. It is instructive, nonetheless, to write the resulting lagrangian at the quadratic level (by again neglecting the decoupled  $V_1$  superfield) and check the Stueckelberg type couplings of gauge fields to the Goldstone modes. The result is

$$\begin{aligned}
S^{(2)} = & -\frac{1}{4} \left( (F_2^{\mu\nu})^2 + (F_3^{\mu\nu})^2 \right) - \frac{1}{2} (F_2^{\mu 5} + mA_3^\mu)^2 - \frac{1}{2} (F_3^{\mu 5} - mA_2^\mu)^2 \\
& - \frac{1}{2} [(\partial_\mu \Sigma_2)^2 + (\partial_\mu \Sigma_3)^2] - \frac{1}{2} (m\Sigma_2 - \partial_5 \Sigma_3)^2 - \frac{1}{2} (m\Sigma_3 + \partial_5 \Sigma_2)^2 \\
& - i \sum_{j=2,3} (\lambda_j \sigma^\mu \partial_\mu \bar{\lambda}_j + \psi_j \sigma^\mu \partial_\mu \bar{\psi}_j) + i [\lambda_2 (\partial_5 + m) \psi_3 - \lambda_3 (\partial_5 + m) \psi_2] . \quad (3.11)
\end{aligned}$$

The Stueckelberg couplings between the gauge field  $A_2^\mu$  and the Goldstone  $A_3^5$  and  $A_3^\mu$  and the Goldstone  $A_2^5$  clearly appear in the first line of (3.11).

By performing the standard KK expansion

$$\Sigma_2 = \frac{1}{\sqrt{\pi L}} \sum_{k=0}^{\infty} r_k \cos \frac{ky}{L} \Sigma_2^{(k)} \quad , \quad \Sigma_3 = \frac{1}{\sqrt{\pi L}} \sum_{k=1}^{\infty} \sin \frac{ky}{L} \Sigma_3^{(k)} \quad , \quad (3.12)$$

we find the KK mass eigenvalues

$$m_{\pm}^{(k)} = \frac{k}{L} \pm m \quad (k \neq 0), \quad m^{(0)} = m. \quad (3.13)$$

The gauge boson and fermion masses of the KK states have a similar structure to those in (3.13).

Notice that in the particular case we were studying in detail the vector multiplets  $V_{2,3}$  can be considered (from the point of view of number of degrees of freedom) as a complex vector multiplet since the vacuum expectation value of  $\phi_1$  gives a mass both to the unbroken  $U(1)$  generated by  $\{\sigma^3/2\}$  and the  $U(1)$  generated by  $\{\sigma^2/2\}$ , already broken by the orbifold boundary conditions. In other cases the counting is a bit more complicated but similar. Take for instance the case where  $SU(3) \rightarrow SU(2) \otimes U(1)$  by the orbifold boundary conditions where the  $SU(3)$  generators  $T_{1,2,3,8}$  are even and  $T_{4,5,6,7}$  are odd. We can further break  $SU(2) \otimes U(1)$  by the Hosotani mechanism when some of the imaginary parts of the scalar components of  $\phi_{4,5,6,7}$ , say  $\phi_4$ , acquire a vacuum expectation value. Only the vector fields along the generators commuting with  $T_4$  will not acquire any mass, namely

$V_4$  and  $V_3 + \sqrt{3}V_8$ . The other six real (three complex) vectors will get a massive spectrum similar to that in (3.13).

## 4. Scherk-Schwarz supersymmetry breaking for the massive vector multiplet

The next step is adding a source of soft supersymmetry breaking. The best suited for our purposes is the generalized dimensional reduction à la Scherk and Schwarz [11], in which various fields are not exactly periodic in the compact coordinate  $x^5$  as in the standard KK reduction, but periodic only up to a symmetry transformation  $U(\omega)$

$$\hat{\phi}(x^5 + 2\pi L) = U(\omega) \hat{\phi}(x^5) , \quad (4.1)$$

where  $\omega$  is a quantized parameter in a string theory context. In an orbifold compactification there are restrictions on the twist matrix  $U$ , namely it has to commute [20] with the orbifold action  $g$ ,  $[U, g] = 0$ . In the particular case of the  $S^1/\mathbb{Z}_2$  compactification, the form of the twist matrix and the consistency condition to be satisfied are [12]

$$U = e^{Mx^5} , \quad \{M, g\} = 0 . \quad (4.2)$$

### 4.1 The case of the hyper Goldstone multiplet

In this case the original  $SU(2) \otimes U(1)_R$  symmetry is spontaneously broken to the  $U(1)_R$  subgroup [2], that contains  $R$ -parity as a discrete subgroup. Therefore the appropriate symmetry to be used in the Scherk-Schwarz reduction in this case is [13] the  $R$ -parity, under which all fermions from the vector and the Goldstone hyper and the bosons from the matter hypermultiplets have odd boundary conditions  $U = -1$ , whereas the rest of fields are periodic. This corresponds to the following Kaluza-Klein fermionic expansion

$$\begin{aligned} (\lambda, \chi_1) &= \frac{1}{\sqrt{\pi L}} \sum_{k=0}^{\infty} r_k \cos \frac{(k+1/2)y}{L} (\lambda^{(k)}, \chi_1^{(k)}) , \\ (\psi_\phi, \chi_2) &= \frac{1}{\sqrt{\pi L}} \sum_{k=1}^{\infty} \sin \frac{(k+1/2)y}{L} (\psi_\phi^{(k)}, \chi_2^{(k)}) , \end{aligned} \quad (4.3)$$

whereas the bosonic KK expansion for the vector and Goldstone hyper is unchanged. The KK bosonic masses are like in (2.6), (2.8), whereas the fermionic masses are shifted accord-

ing to

$$m_f^{(k)^2} = m^2 + \frac{(k + 1/2)^2}{L^2} . \quad (4.4)$$

## 4.2 The case of the vector Goldstone multiplet

In this case the original symmetry  $SU(2) \otimes U(1)_R$  symmetry is spontaneously broken to  $SU(2)_R$  [2], and the appropriate twist matrix which satisfies the consistency requirement (4.2) is the  $U(1)$  subgroup of the  $SU(2)_R$   $R$ -symmetry in five dimensions [12], which acts on the fermions of the two vector multiplets as

$$\begin{aligned} \begin{pmatrix} \hat{\lambda}_3 \\ \hat{\psi}_3 \end{pmatrix} &\equiv \begin{pmatrix} \cos(\omega y/L) & -\sin(\omega y/L) \\ \sin(\omega y/L) & \cos(\omega y/L) \end{pmatrix} \begin{pmatrix} \lambda_3 \\ \psi_3 \end{pmatrix} , \\ \begin{pmatrix} \hat{\lambda}_2 \\ \hat{\psi}_2 \end{pmatrix} &\equiv \begin{pmatrix} \cos(\omega y/L) & \sin(\omega y/L) \\ -\sin(\omega y/L) & \cos(\omega y/L) \end{pmatrix} \begin{pmatrix} \lambda_2 \\ \psi_2 \end{pmatrix} , \end{aligned} \quad (4.5)$$

where  $\omega = 1/2$  in the simplest examples worked out in the literature. For a general  $\omega$ , the fermionic spectrum (eigenvalues of the fermionic mass matrix) is shifted according to

$$m_{f\pm}^{(k)} = \frac{k \pm (\omega + mL)}{L} \quad (4.6)$$

In the particular case  $\omega + mL = 1/2$  the up and the down mass shifts in (4.6) are equivalent when summing over the whole KK tower of states.

## 5. One-loop vacuum energy and radion stabilization

In this section we will compute the one-loop effective potential in the background of the radion field. We will consider, on top of the gravitational contribution a number of massless ( $N_v$  vectors and  $N_h$  hypers) and massive ( $N_H$  hypers and  $N_V$  vectors) multiplets<sup>3</sup>. We will assume that supersymmetry is broken by the SS mechanism with an arbitrary parameter  $\omega$  that should be fixed to the value  $1/2$  if the Goldstones come from hypermultiplets.

The formalism of massive vector multiplets was presented in previous sections. For completeness we will now consider the case of hypermultiplets with a supersymmetric mass  $M$ . Let us then consider the hypermultiplet  $(\chi_1, \chi_2) \equiv (\chi_1, \chi_2; \psi; F_1, F_2)$ , with parities

---

<sup>3</sup>We are using a compact notation in order to cover both cases of Sections 2 and 3. For the case of the hyper Goldstone of Section 2,  $N_V$  is actually twice the number of the vector multiplets, whereas  $N_V$  is equal to the number of vector multiplets for the case of vector Goldstone of Section 3.

$(+, -)$  and let us introduce the odd mass  $M(x^5) = \epsilon(x^5)M$ , where  $\epsilon(x^5)$  is the sign function. The supersymmetric lagrangian is given in  $\mathcal{N} = 1$  language by [5, 6]

$$\mathcal{L} = \int d^4\theta [\bar{\chi}_1\chi_1 + \bar{\chi}_2\chi_2] - \left\{ \int d^2\theta\chi_1 [\partial_5 - M(x^5)] \chi_2 + \text{h.c.} \right\} \quad (5.1)$$

or in components

$$\begin{aligned} \mathcal{L} = & -|\partial_M\chi_1|^2 - |\partial_M\chi_2|^2 - i\bar{\psi}\gamma^M\partial_M\psi + M(x^5)\bar{\psi}\psi \\ & - [M^2 + \partial_5 M(x^5)]|\chi_1|^2 - [M^2 - \partial_5 M(x^5)]|\chi_2|^2 \end{aligned} \quad (5.2)$$

The mass eigenvalues corresponding to the system described by the lagrangian (5.2) were computed in Ref. [17]. In particular the eigenvalues of the fermionic mass matrix are given by

$$m_k^2 = \frac{k^2}{L^2} + M^2(1 - \delta_{k0}) \quad (5.3)$$

and those of the bosonic mass matrix by the solution to the equation

$$\sin^2(\pi\omega) = \frac{m_k^2}{\Omega_k^2} \sin^2(\Omega_k\pi L) \quad (5.4)$$

where  $\Omega_k = \sqrt{m_k^2 - M^2}$ .

Up to this point we have parametrized the theory as a function of the interval length scale  $L$ . We will now introduce the physical radion field  $R$ . In order to do that we will parametrize the 5D metric in the Einstein frame in terms of the “dilaton” field  $\varphi$  as [14]

$$ds^2 = \varphi^{-1/3} g_{\mu\nu} dx^\mu dx^\nu + \varphi^{2/3} dx^5 dx^5 \quad (5.5)$$

where  $x^5$  goes from 0 to  $\pi L$ . The radion field whose VEV determines the size of the extra dimension is the physical radius given by  $R = \varphi^{1/3}L$ . The length  $L$  is unphysical. It will drop out once the VEV of the radion  $\varphi_c^{1/3}L$  is fixed and the effective 4D theory will only depend on  $R$ . Notice that in previous sections we were identifying  $R$  with  $L$  in units of  $\varphi_c$ . From here on we will turn on the explicit dependence on  $\varphi_c$ . Turning on a value of  $\varphi_c \neq 1$  in the action amounts (when defining canonically normalized fields  $\tilde{\phi} = \varphi_c^{1/6}\phi$ ) to changing  $L \rightarrow \varphi_c^{1/2}L$ ,  $M \rightarrow \varphi_c^{-1/6}M$  such that  $LM \rightarrow \varphi_c^{1/3}LM = RM$ . All the previously considered spectra scale correspondingly with  $\varphi_c$  as we will see now.

We will first consider the massless sector, i.e. the gravitational sector plus  $N_v$  massless vector multiplets and  $N_h$  massless hypermultiplets. The mass squared of KK modes of the

gravitino, gauginos and hyperscalars is given by:

$$m_k^2 = \frac{1}{\varphi_c} \frac{(k + \omega)^2}{L^2} \quad (5.6)$$

and those of the rest of KK modes is given by (5.6) after fixing  $\omega = 0$ . The Coleman-Weinberg one-loop effective potential [15] can be easily computed to be [16]

$$V_{\text{eff}} = \frac{2 + N_v - N_h}{64\pi^6 \varphi_c^2 L^4} [3Li_5(r) + 3Li_5(1/r) - 6\zeta(5)] \quad (5.7)$$

where  $r = \exp(2\pi i\omega)$ , the polylogarithms are defined as

$$Li_n(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^n} \quad (5.8)$$

and  $\zeta(n) = Li_n(1)$ .

The effective potential of  $N_H$  massive hypermultiplets with a common supersymmetric mass  $M_H$  was computed in Ref. [17]. In the presence of a SS twist  $\omega$  the effective potential is given by

$$V_{\text{eff}} = \frac{N_H}{64\pi^6 \phi_c^2 L^4} g(x_H, \omega) \quad (5.9)$$

where  $x_H = M_H \pi R$  and

$$g(x, \omega) = 8 \int_0^\infty dz z^3 \log \left[ 1 + \frac{(z^2 + x^2) \sin^2(\pi\omega)}{z^2 \sinh^2(\sqrt{z^2 + x^2})} \right] \quad (5.10)$$

Finally the mass eigenvalues of fermions in gauge multiplets with supersymmetric mass  $M_V$  and where the Goldstone bosons are in hypermultiplets is given by [Eq. (4.4)]

$$m_k^2 = \frac{M_V^2}{\varphi_c^{1/3}} + \frac{1}{\varphi_c} \frac{(k \pm \omega)^2}{L^2} \quad (5.11)$$

and the effective potential is given by

$$V_{\text{eff}} = \frac{N_V}{64\pi^6 \varphi_c^2 L^4} f(x_V, \omega) \quad (5.12)$$

where  $x_V = M_V \pi R$  and

$$\begin{aligned} f(x, \omega) = & 4x^2 Li_3(re^{-2x}) + 6x Li_4(re^{-2x}) + 3Li_5(re^{-2x}) + \text{h.c.} \\ & - 8x^2 Li_3(e^{-2x}) - 12x Li_4(e^{-2x}) - 6Li_5(e^{-2x}) \end{aligned} \quad (5.13)$$

Notice that here, as we already explained in the previous section,  $\omega = 1/2$ .

Similarly the mass eigenvalues of fermions in complex gauge multiplets with supersymmetric mass  $M_V$  and where the Goldstone bosons are in gauge multiplets (Hosotani breaking) is given by [Eq. (4.6)]

$$m_k^2 = \frac{1}{\varphi_c} \left( \frac{k \pm (\omega + M_V R)}{L} \right)^2 \quad (5.14)$$

and its contribution to the effective potential is given by Eq. (5.12) with the function  $f(x, \omega)$  defined by

$$f(x, \omega) = 3Li_5(r e^{2ix}) - 3Li_5(e^{2ix}) + \text{h.c.} \quad (5.15)$$

As it is clear from (5.15) the potential from massive gauge bosons when the gauge symmetry is broken by the Hosotani mechanism is oscillatory and therefore essentially different to the case where Goldstone bosons are in hypermultiplets.

We will first refer to the case where Goldstone bosons are in hypermultiplets and therefore supersymmetry breaking is achieved with a Scherk-Schwarz parameter  $\omega = 1/2$ . In Ref. [18] a radion potential with anti de Sitter minimum was created by the presence of  $N_H$  hypermultiplets with a common supersymmetric mass. The conditions for such a behaviour,  $V_{\text{eff}} \rightarrow +\infty$  when  $R \rightarrow 0^+$  and  $V_{\text{eff}} \rightarrow 0^-$  when  $R \rightarrow \infty$ , were fulfilled if  $2 + N_v - N_h - N_H < 0$ ,  $2 + N_v - N_h > 0$ . We now want a radion potential exhibiting a minimum with positive (de Sitter) or zero (Minkowski) vacuum energy. A necessary condition for that is that the potential  $V_{\text{eff}} \rightarrow +\infty$  when  $R \rightarrow 0^+$  and  $V_{\text{eff}} \rightarrow 0^+$  when  $R \rightarrow \infty$ . These conditions are fulfilled provided that

$$2 + N_v + N_V - N_h - N_H < 0, \quad 2 + N_v - N_h < 0 \quad (5.16)$$

respectively. Then we can write the effective potential as

$$V_{\text{eff}} = \frac{N_h - 2 - N_v}{64} M_H^6 L^2 \mathcal{V}(x), \quad (5.17)$$

where  $x \equiv x_H = \pi M_H R$  and

$$\mathcal{V}(x) = \frac{1}{x^6} \{-f(0, \omega) + \delta_H g(x, \omega) + \delta_V f(\alpha x, \omega)\} \equiv \frac{1}{x^6} v(x) \quad (5.18)$$

We have defined in (5.18)

$$\delta_{H,V} = \frac{N_{H,V}}{N_h - 2 - N_v} \quad (5.19)$$

and  $\alpha = M_V/M_H$ .

It is easy to see that the minimum condition with zero vacuum energy  $\mathcal{V}'(x) = \mathcal{V}(x) = 0$  is equivalent to the simplest one  $v'(x) = v(x) = 0$  and that a necessary condition for de

Sitter or Minkowski minimum is  $M_V < M_H$ , i.e.  $\alpha < 1$ . In fact it is easy to work out cases where this happens. A simple example is provided in Fig. 1 where we have considered the case  $\delta_H = \delta_V = 1$  which corresponds to  $N_H = N_V = 2(N_h - N_v - 2)$ . The upper (thin) curve exhibits a potential where the minimum is de Sitter: it corresponds to  $\alpha = 0.27$ . In the lower (thick) curve the value of  $\alpha$  has been fine-tuned to have a zero vacuum energy (Minkowski minimum). The minimum is located at  $x \simeq 3.1$  which corresponds to values of the supersymmetric masses,  $M_H \simeq 1/R$  and  $M_V \simeq 0.27/R$ . Of course other similar cases can easily be found.

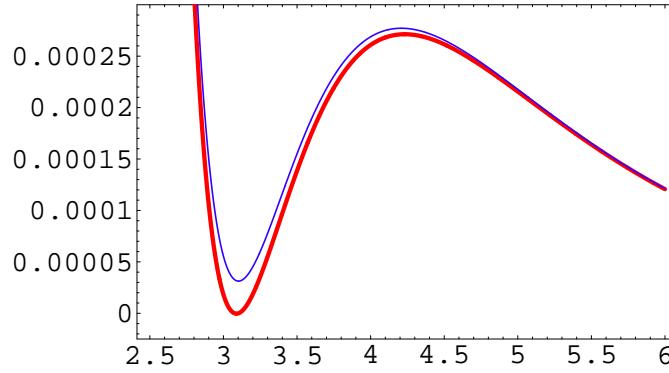


Figure 1: *Radion effective potential  $\mathcal{V}(x)$  as defined in the text, with Goldstone bosons in hypermultiplets, for  $\delta_H = \delta_V = 1$  and  $\omega = 1/2$ . Upper (blue-thin) potential has de Sitter minimum: it is for  $\alpha = 0.27$ . Lower (red-thick) potential with Minkowski minimum:  $\alpha \simeq 0.27$  is tuned to zero vacuum energy.*

In the case where Goldstone bosons are in gauge multiplets, and the breaking of the gauge symmetry is by the Hosotani mechanism, the potential from massive gauge bosons is an oscillatory function as given in Eq. (5.15). In this case it is also possible to tune the parameters such that there is a Minkowski minimum for the function  $v(x)$  in (5.18) at  $x = x_0$  and other AdS minima for higher values of  $x$ . However since the whole effective potential is  $\propto v(x)/x^6$  the Minkowski minimum remains and AdS minima are suppressed by the factor  $1/x^6$ . On the other hand the tunnelling probability to the AdS minima is exponentially small and the Minkowski vacuum is essentially stable on cosmological times [18]. A particular example is presented in Fig. 2 where we have considered the case  $2\delta_H = 3\delta_V = 3$ , i.e.  $2N_H = 3N_V = 3(N_h - 2 - N_v)$ , a Scherk-Schwarz parameter  $\omega = 1/2$

and  $\alpha \simeq 0.39$ . The minimum is located at  $x \simeq 8.1$  which corresponds to values of the supersymmetric masses  $M_H \simeq 2.6/R$  and  $M_V \simeq 1/R$ .

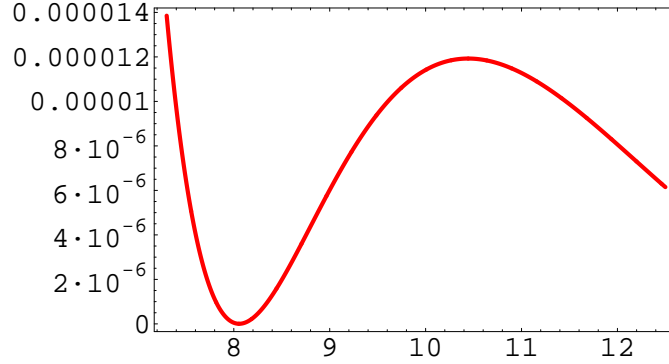


Figure 2: *Radion effective potential for Goldstone bosons in gauge multiplets for  $2\delta_H = 3\delta_V = 3$ ,  $\omega = 1/2$  and  $\alpha \simeq 0.39$ .*

Finally it is also useful to present a four-dimensional description of the stabilization mechanism in terms of the standard 4D supergravity lagrangian [19]. This description is possible for large radii where one can truncate the higher derivative corrections to the effective action. The radius  $R$  and the fifth component of the gauge field in the gravitational multiplet  $(g_{MN}, A_M, \Psi_M)$  form a complex field  $T$  with bosonic components  $T = t + iA_5$ . By ignoring the one-loop corrections, it is well-known that the effective lagrangian describing the radion with Scherk-Schwarz supersymmetry breaking is of no-scale type with a constant superpotential term [20, 12, 6, 21]. The one-loop corrections produce a deformation of the no-scale structure in the Kahler potential. We will use the relation between the 5D fundamental Planck scale  $M_5$ , the 4D Planck mass  $M_P$  and the fifth radius  $R$  via

$$RM_5^3 = M_P^2 \quad , \quad R = tM_5^{-1} \quad , \quad (5.20)$$

We can parametrize the one-loop deformation of the no-scale structure as <sup>4</sup>

$$K = -3 \log(T + \bar{T}) - \Delta K, \quad \Delta K \equiv \frac{\Delta}{T + \bar{T}} \quad (5.21)$$

---

<sup>4</sup>In what follows we will use units where  $M_P \equiv 1$ .



In the presence of a constant superpotential  $W = \omega$  and in the large radius limit we can relate  $\Delta K$  with the one-loop effective potential  $V$  as

$$V = e^K (K^{T\bar{T}} |D_T W|^2 - 3|W|^2) = \frac{|\omega|^2}{(T + \bar{T})^2} \Delta'' \quad (5.22)$$

where  $\Delta'' = \partial^2 \Delta / \partial(T + \bar{T})^2$ .

We can now express the 4D effective supergravity Kahler potential in the presence of massless hypers and massive hypers and vector multiplets. For the case where Goldstone bosons come in hypermultiplets it is given by as

$$\Delta = \frac{1}{(T + \bar{T})^2} \left[ \alpha_0 + \alpha_1 (T + \bar{T}) e^{-\beta_1 (T + \bar{T})} + \alpha_2 e^{-\beta_2 (T + \bar{T})} \right] + \dots, \quad (5.23)$$

where the dots represent subdominant terms in the large radius limit. The scalar potential in the large radius limit  $T + \bar{T} \gg 1$  is given by

$$V = \frac{|\omega|^2}{(T + \bar{T})^3} \left[ \frac{6\alpha_0}{(T + \bar{T})^3} + \alpha_1 \beta_1^2 e^{-\beta_1 (T + \bar{T})} + \frac{\alpha_2}{(T + \bar{T})} \beta_2^2 e^{-\beta_2 (T + \bar{T})} + \dots \right]. \quad (5.24)$$

By using (5.20), we find that the scalar potential (5.24) agrees with the leading contribution to the vacuum energy (5.17) in the large radius approximation for

$$\begin{aligned} \alpha_0 &\sim \frac{-2(N_h - 2 - N_v)}{\omega^2} \sum_{k=1}^{\infty} \frac{\sin^2(\pi \omega k)}{k^5}, \quad \alpha_1 \sim N_H, \quad \alpha_2 \sim \frac{-8 \sin^2(\pi \omega)}{\omega^2} N_V, \\ \beta_1 &\sim \pi M_H, \quad \beta_2 \sim \pi M_V. \end{aligned} \quad (5.25)$$

The main reason in getting the possibility of having positive (de Sitter) or zero vacuum energy is the oppositive sign contribution of the hypers (coefficient  $\alpha_1$ ) and of the vectors (coefficient  $\alpha_2$ ) in the one-loop vacuum energy (5.24). Finally notice that the functional form of (5.24) is similar to scalar potential in racetrack models of gaugino condensation [22].

For the case where Goldstone bosons absorbed by massive gauge multiplets come in vector multiplets (5.23) should be replaced in the large radius limit by

$$\begin{aligned} \Delta &= \frac{1}{(T + \bar{T})^2} \left\{ \alpha_0 + \alpha_1 (T + \bar{T}) e^{-\beta_1 (T + \bar{T})} \right. \\ &\quad \left. + \frac{\alpha_3}{(T + \bar{T})^2} \left[ 3 Li_7 \left( e^{2i\pi\omega + i\beta_3 (T + \bar{T})} \right) - 3 Li_7 \left( e^{i\beta_3 (T + \bar{T})} \right) + \text{h.c.} \right] \right\}, \end{aligned} \quad (5.26)$$

where

$$\alpha_3 \sim -\frac{N_V}{\omega^2 M_V^2}, \quad \beta_3 \sim \pi M_V. \quad (5.27)$$

Notice that the oscillatory behaviour of the potential is transmitted to the Kahler potential.

## 6. Conclusions

The goal of the present paper was to show that, generalizing Ref. [1] by including massive five-dimensional vector multiplets together with massive 5D hypermultiplets of different masses and combining them with supersymmetry breaking á la Scherk-Schwarz, quantum loops generate a radion potential which can stabilize it at a positive or zero vacuum energy. There are two different ways of Higgsing a vector multiplet in 5D, by absorbing either a hyper Goldstone, or by absorbing the fifth component of another gauge field of appropriate parity. In an orbifold compactification the second case asks for a non-abelian gauge structure, generate a spontaneous breaking of the 5D Lorentz invariance and is equivalent to the Hosotani mechanism of gauge symmetry breaking. The four-dimensional description of this phenomenon, valid at large radii, involves a one-loop modification of the Kahler potential, which contains two (or more) exponential terms coming with different signs. We would like to comment here on the differences between the stabilisation presented here and the one put forward in [23] in the context of flux compactifications. The authors of [23] use fluxes in order to generate a constant in the superpotential, in combination with nonperturbative effects like D3 instantons or gaugino condensation on D7 branes in order to produce a non-perturbative superpotential for the radion. A stabilisation at large radius in their case need a certain amount of tuning between several fluxes in order to generate a small constant in the superpotential. The present mechanism does not need any particularly small numbers or nonperturbative effects, whereas massive vector multiplets and hypermultiplets are generally present in various string compactifications. For a qualitatively different proposal of the role of the massive vectors in string theory, see also [24].

There are several possible generalizations of our work. First of all, a full 5D supergravity description of the two different realizations of the Higgs phenomenon would be very useful for further studies and for a more detailed analysis of the 4D supergravity description of the stabilization mechanism of the Minkowski and de Sitter solutions. Secondly, it would be interesting to combine, in the framework of string models, the present stabilization with other mechanisms of moduli stabilisation present in the literature in order to find realistic models with all moduli stabilized.

## Acknowledgments

This work is supported in part by the CNRS PICS no. 2530 and 3059, INTAS grant 03-51-6346, the RTN grants MRTN-CT-2004-503369, MRTN-CT-2004-005104, by a European Union Excellence Grant, MEXT-CT-2003-509661 and partly by CICYT, Spain, under contracts FPA 2004-02015 and FPA 2002-00748 and by the CICYT/IN2P3 grant IN2P3 05-19. We are grateful to P. Fayet and A. Pomarol for very useful discussions. ED would like to acknowledge the hospitality of the theory group of IFAE, Autonomous Univ. of Barcelona during the completion of this work.

## References

- [1] E. Ponton and E. Poppitz, JHEP **0106** (2001) 019 [arXiv:hep-ph/0105021]; M. A. Luty and N. Okada, JHEP **0304** (2003) 050 [arXiv:hep-th/0209178].
- [2] P. Fayet, Nucl. Phys. B **149** (1979) 137.
- [3] Y. Hosotani, Phys. Lett. B **126** (1983) 309; Y. Hosotani, Phys. Lett. B **129** (1983) 193; Y. Hosotani, Annals Phys. **190** (1989) 233.
- [4] N. Marcus, A. Sagnotti and W. Siegel, Nucl. Phys. B **224** (1983) 159.
- [5] N. Arkani-Hamed, T. Gregoire and J. Wacker, JHEP **0203** (2002) 055 [arXiv:hep-th/0101233].
- [6] D. Marti and A. Pomarol, Phys. Rev. D **64** (2001) 105025 [arXiv:hep-th/0106256].
- [7] N. Arkani-Hamed, A. G. Cohen and H. Georgi, Phys. Lett. B **516** (2001) 395 [arXiv:hep-th/0103135]; C. A. Scrucca, M. Serone, L. Silvestrini and F. Zwirner, Phys. Lett. B **525**, 169 (2002) [arXiv:hep-th/0110073]; R. Barbieri, R. Contino, P. Creminelli, R. Rattazzi and C. A. Scrucca, Phys. Rev. D **66** (2002) 024025 [arXiv:hep-th/0203039]; G. von Gersdorff and M. Quiros, Phys. Rev. D **68** (2003) 105002 [arXiv:hep-th/0305024].
- [8] E. Dudas, T. Gherghetta and S. Groot Nibbelink, Phys. Rev. D **70** (2004) 086012 [arXiv:hep-th/0404094].

- [9] E.C.G Stueckelberg, *Helv. Phys. Acta* **11** (1938) 225; V.I Ogievetskii and I.V Polubarinov, *JETP* **14** (1962) 179.
- [10] For a review see e.g.: M. Quirós, “New ideas in symmetry breaking,” [arXiv:hep-ph/0302189](#).
- [11] J. Scherk and J. H. Schwarz, *Nucl. Phys. B* **153** (1979) 61; E. Cremmer, J. Scherk and J. H. Schwarz, *Phys. Lett. B* **84** (1979) 83; P. Fayet, *Phys. Lett. B* **159** (1985) 121.
- [12] E. Dudas and C. Grojean, *Nucl. Phys. B* **507** (1997) 553 [[arXiv:hep-th/9704177](#)]; I. Antoniadis and M. Quiros, *Phys. Lett. B* **416** (1998) 327 [[arXiv:hep-th/9707208](#)].
- [13] R. Barbieri, L. J. Hall and Y. Nomura, *Phys. Rev. D* **63** (2001) 105007 [[arXiv:hep-ph/0011311](#)].
- [14] T. Appelquist and A. Chodos, *Phys. Rev. Lett.* **50** (1983) 141; T. Appelquist and A. Chodos, *Phys. Rev. D* **28** (1983) 772.
- [15] S. R. Coleman and E. Weinberg, *Phys. Rev. D* **7** (1973) 1888.
- [16] A. Delgado, A. Pomarol and M. Quiros, *Phys. Rev. D* **60** (1999) 095008 [[arXiv:hep-ph/9812489](#)].
- [17] G. von Gersdorff, L. Pilo, M. Quiros, D. A. J. Rayner and A. Riotto, *Phys. Lett. B* **580** (2004) 93 [[arXiv:hep-ph/0305218](#)].
- [18] G. von Gersdorff, M. Quiros and A. Riotto, *Nucl. Phys. B* **689** (2004) 76 [[arXiv:hep-th/0310190](#)].
- [19] E. Cremmer, S. Ferrara, L. Girardello and A. Van Proeyen, *Nucl. Phys. B* **212** (1983) 413; E. Cremmer, B. Julia, J. Scherk, S. Ferrara, L. Girardello and P. van Nieuwenhuizen, *Nucl. Phys. B* **147** (1979) 105.
- [20] S. Ferrara, C. Kounnas, M. Porrati and F. Zwirner, *Nucl. Phys. B* **318** (1989) 75.
- [21] D. E. Kaplan and N. Weiner, [arXiv:hep-ph/0108001](#); G. von Gersdorff and M. Quiros, *Phys. Rev. D* **65** (2002) 064016 [[arXiv:hep-th/0110132](#)]; J. A. Bagger, F. Feruglio and F. Zwirner, *Phys. Rev. Lett.* **88** (2002) 101601 [[arXiv:hep-th/0107128](#)].
- [22] N. V. Krasnikov, *Phys. Lett. B* **193** (1987) 37; T. R. Taylor, *Phys. Lett. B* **252** (1990) 59; J. A. Casas, Z. Lalak, C. Munoz and G. G. Ross, *Nucl. Phys. B* **347** (1990) 243.

[23] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, Phys. Rev. D **68** (2003) 046005 [arXiv:hep-th/0301240]; C. P. Burgess, R. Kallosh and F. Quevedo, JHEP **0310** (2003) 056 [arXiv:hep-th/0309187].

[24] S. Watson, Phys. Rev. D **70** (2004) 066005 [arXiv:hep-th/0404177].